Exercise Sheet 2

Exercise 2.1. Let $x \in \mathbb{F}_2^n$ be of weight d. What is the number of binary vectors of weight w that are orthogonal to x? (*Hint:* Use MacWilliams identities.)

Exercise 2.2. Let d be an odd positive integer. Show that there is a $(n, k, d)_2$-code iff there is an $(n + 1, k, d + 1)_2$-code.

Exercise 2.3. Let $A_q(n, d)$ be the maximum k for which an $[n, k, d]_q$-code exists. Show that $A_2(n, 2) = n - 1$.

Exercise 2.4. The *extended Hamming code* is constructed as follows: start with the $[7, 4, 3]_2$-Hamming code and add a position to each codeword. In that position, put a 1 if the codeword is of odd weight, and put a 0 otherwise.

1. Show that the extended Hamming code is an $[8, 4, 4]_2$-code and calculate a generator and a check matrix for this code.
2. Show that the dual of the extended Hamming code is equal to the code itself.

Exercise 2.5. Let $\mathcal{C}$ be an $[n, k]$ code over $\mathbb{F}_q$.

1. Show that the minimum distance of $\mathcal{C}$ is the largest integer d such that every $k \times (n - d + 1)$ submatrix of its generator matrix has rank k.
2. Show that $\mathcal{C}$ is MDS if and only if its dual is.