Polar Source Codes for Non-binary Sources

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Abstract

The abstract goes here.

Index Terms

keywords

I. Introduction

II. Source Coding with a Fidelity Criterion

Let U denote a discrete memoryless source taking values in a source alphabet $\mathcal{U}$. The distribution of the source is defined by a pmf f_U. We want to represent U by a destination alphabet $\mathcal{V}$. As a random variable we denote the destination by V. Without loss of generality we assume that $U, V \in \mathbb{R}$. The error made in representing U by V is captured by a non-negative distortion measure $d(u, v)$

$$d : \mathcal{U} \times \mathcal{V} \mapsto \mathbb{R}. \tag{1}$$

A joint probability distribution on (U, V) is defined by a channel probability density $f_{V|U}$. The rate distortion function is defined by

$$R(d*) = \min_{f_{V|U}: d<d*} I(U; V). \tag{2}$$

Once a $f_{V|U}$ is found we can compute a backward testchannel $f_{U|V}$.

A. The Gaussian Channel

We assume that the source U is symmetric which implies that for each $u \in \mathcal{U}$ there exists a $-u \in \mathcal{U}$ such that $f_U(u) = f_U(-u)$. In this case we should be able to use a symmetric destination alphabet and the test channel should posses some symmetry $f_{V|U}(v|u) = f_{V|U}(-v| - u)$. This we have to proof.

We assume that $|\mathcal{V}| = 2^m$ for some integer m and associate with each element of $\mathcal{V}$ an element of $\mathbb{F}_2^m$. In this way we write a $v \in \mathcal{V}$ as a tuple $v_1, \ldots, v_m$ where each of the $v_i \in \mathbb{F}_2$. By the chain rule of information we have

$$I(U; V) = I(U; (V_1, \ldots, V_m)) = I(U; V_1) + I(U; V_2|V_1) + \cdots + I(U; V_m|V_1, \ldots, V_{m-1}) \tag{3}$$

Now consider the second term on the righthandside which we can write as

$$I(U; V_2|V_1) = E\left[\frac{p(U|V_1, V_2)}{\sum_{v_2'} P(U|V_1, V_2 = v_2')P(V_2 = v_2'|V_1)}\right] \tag{4}$$

Note that $P(V_2|V_1)$ is not uniform and depends on the value of V_1. However, we do have that $P(V_2|V_1 + 1) = P(V_2 + 1|V_1)$.

B. Sufficient statistic and log-likelihood ratios

For analysis and decoding algorithms we use the log-likelihood ratio. For (4) the corresponding LLR is defined as

$$L = \log \frac{p(V_2 = 0|U, V_1)}{p(V_2 = 1|U, V_1)} = \log \frac{p(U|V_1, V_2 = 0)}{p(U|V_1, V_2 = 0)} + \log \frac{p(V_2 = 0|V_1)}{p(V_2 = 1|V_1)} \tag{5}$$

Note that we have the following symmetries

$$p(v_2|v_1) = p(v_2 + 1|v_1 + 1), \tag{6}$$

and

$$p(u|v_1, v_2 = 0) = p(-u|v_1 + 1, v_2 = 1). \tag{7}$$

With this symmetry we can show that as a random variable the LLR satisfies $L(-u, v_1 + 1) = -L(u, v_1)$. Furthermore, the L density will be symmetric?? However, when we compute capacity we can not use the funtional $E[log_2(1 + \exp(-L)]$ since the prior is not uniform. We have to look at this and the channel equivalence lemma.

III. Conclusion

A little more analysis required.