Neuroscience meets Coding theory!

Neural Associative Memory with Exponential Pattern Retrieval Capacity

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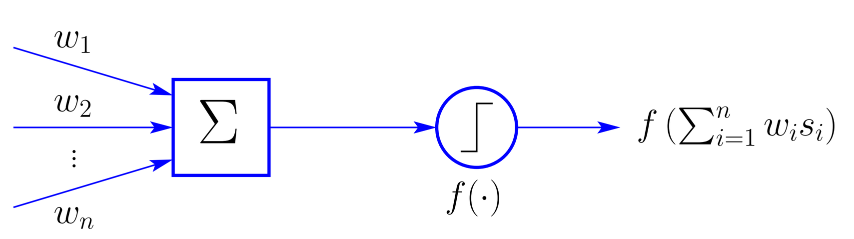
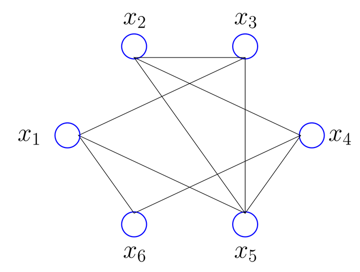
1. Introduction

Associative memory problem : Find the closest stored vector (in Hamming distance) to a given query vector.

Neural network implementation:





- Graph edge weights are chosen intelligently in order to perform association.
- Recall procedure is iterative and relies on simple neural operations.

2. Problem formulation

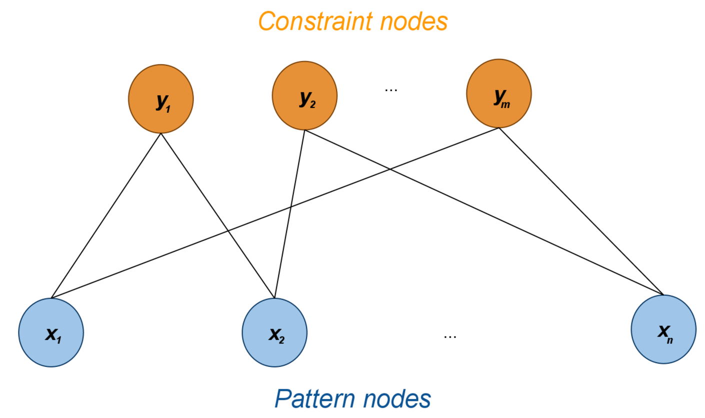
Maximize the number of stored patterns while ensuring some noise tolerance

- Extensively studied over the past few decades [1], [2], [5].
- *Low in storage capacity*: Storage capacity has been shown to be at best equal to n, the number of neurons, *when required to memorize purely random patterns* [1].
- Recently, some works have been done to improve the storage capacity by memorizing *structured patterns* (see [3, 4] and references therein).
- Capacity achieving error correcting codes:
  - Similar function, recall the closest codeword from a noisy observation
  - Shannon: Exponential number of codewords can be decoded!

We introduce an associative memory framework with exponential storage capacity based on a network of non-binary neurons

3. The proposed framework

- We utilize a bipartite network of n pattern neurons and m constraint neurons.
- The state of each neuron indicates its short-term firing rate and is a bounded non-negative integer.
- The connectivity of the network is determined by the adjacency matrix H.
- The elements of H, i.e. the connection weights, are either 0 or 1.



- The state of pattern node j, denoted by x_j, can be any bounded non-negative integer.
- The state of constraint node i, denoted by y_i, can be 1, -1 or 0.
- Each constraint node y_i has a decision threshold b_i.
- For a particular H and a vector of decision thresholds b, enumerate the set of pattern node states x such that $Hx = b$

Hence, instead of *memorizing* randomly picked sequences of length n, we store only those that satisfy m constraints.

4. The association process

The proposed framework finds the closest stored pattern to the probe $\hat{x}$ via forward and backward iterations.

Forward iteration

- Constraint nodes decide their state based on simple *neural* operations:

$$y_i = \begin{cases} 1, & h_i < b_i \\ 0, & h_i = b_i \\ -1, & \text{otherwise} \end{cases}.$$

where $h_i = \sum_{j=1}^{n} H_{ij} x_j$,

Backward iteration

- Each pattern node j computes the quantity

$$g_j = \frac{\sum_{i=1}^{m} H_{ij} y_i}{d_p}.$$

The sign of g_j is an indication of the sign of the noise that affects x_j, and $|g_j|$ indicates the confidence level in the decision.

- The state of pattern node j is updated using either of the following two strategies:
  1. *Winner-take-all* strategy: only the node with the maximum $|g_j|$ is updated.
  2. *Bit-flipping* strategy: all pattern nodes with $|g_j| \geq \gamma$ are updated based on the sign of g_j.

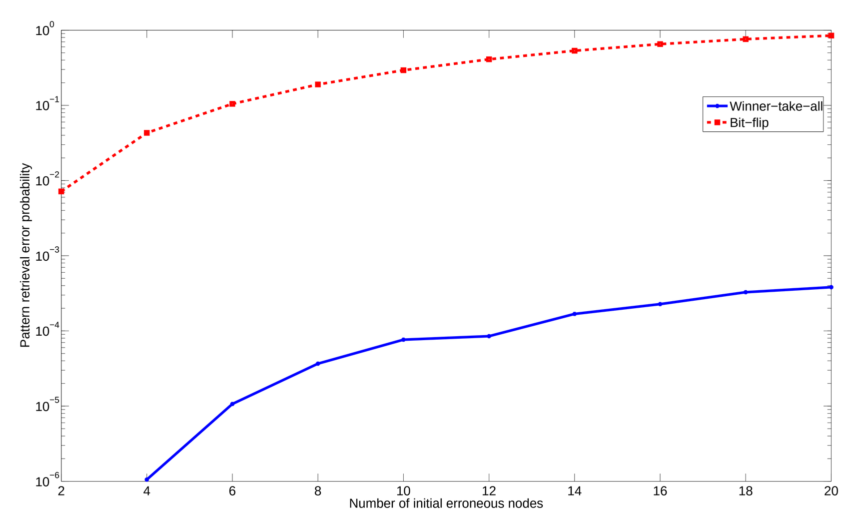
5. Results

Theoretical results

- Connectivity properties of H should be chosen carefully.
- Utilize framework of *expander graphs* for this purpose.
- The proposed framework is guaranteed to correct two erroneous nodes [4].
- For proper choice of row degrees in the constraint matrix, our scheme admits an exponential storage capacity in terms of n.

Numerical results

- The graph below illustrates the pattern retrieval error rate vs. the number of initial erroneous nodes.



6. Some remarks

More details about our approach and theoretical results can be found in [4].

Future extensions:

- Learning constraints from data
- Finding a simple map from a set of purely random patterns to those with some structure that is biologically meaningful.

Acknowledgments

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References

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