Progress Report
16-30 June 2012

Amin Karbasi
E-mail: amin,karbasi@epfl.ch
Amir Hesam Salavati
E-mail: hesam.salavati@epfl.ch

Supervisor: Prof. Amin Shokrollahi
E-mail: amin.shokrollahi@epfl.ch

Algorithmics Laboratory (ALGO)
Ecole Polytechnique Federale de Lausanne (EPFL)

October 26, 2012

1 Summary

In the past two weeks, we were busy applying the analysis proposed for spatially-coupled codes to our model. To this end, we read another paper about this method and worked on designing proper simulation scenarios to test this approach on a database of natural images. The set of images is perfect for the spatially-coupled model since we can divide the picture into overlapping clusters and learn the constraints for each cluster. Then, each cluster in a row will act as a component code in a Generalized LDPC (GLDPC) code. The clusters in one row of the image then form a (GLDPC) code and different rows will form a spatially coupled GLDPC code, much like the approach proposed in [1]. This work will hopefully form the basis for a paper which we would like to submit to Allerton 2012.

2 Approaching Capacity at High Rates with Iterative Hard-Decision Decoding [1]

In [1] the authors propose a novel spatially-coupled Generalized LDPC (GLDPC) code which can achieve capacity at very high rates. The GLDPC code uses component codes that can correct up to t errors, using hard-decision decoding techniques. An ensemble of these GLPDC codes are then coupled together and it is shown that the spatially-coupled code will achieve capacity for very high coding rates (actually the rate is very close to 1). The bounds on the achievability threshold of the channel are derived using the novel analysis technique suggested in [2], which uses the notion of potential to analyze the density evolution algorithm. As mentioned above, the proposed model is very similar to our neural works. Therefore, the method used to analyze the code could also be helpful for our work.

The main contribution of this paper is a spatially coupled GLDPC which can be used to achieve capacity over BSC channels at very high rates. Roughly speaking, the authors show that their proposed code can achieve capacity in a BSC channel with error probability of $2t/(2^\nu - 1)$ for some fixed t and ν with a spatially coupled GLDPC code with rate $1 - 2\nu t/(2^\nu - 1)$.

To go into more details, let $\mathcal{C}$ be a linear (n, k, d) code that correct up to t bits of error (i.e. $2t + 1 \leq d$). Now we use these codes as components of a Generalized LDPC (GLDPC) code as follows: we will consider m such components each of which is connected to n variable nodes. In total, we will have $N = nm/2$ variable nodes, which mean each variable node is connected to exactly two component codes. So in the Tanner graph, we have N variable nodes of degree 2 and m component code of degree n.

At the component codes, the authors use Bounded-Distance Decoding (BDD) techniques, which is much less complex than the MAP decoding. This decoder is given by equation (1), in which $\mathbf{v} \in \{0, 1\}^n$ is the received binary vector and $D_i(\mathbf{v})$ is the i^{th} decoded bit.

$$D_i(\mathbf{v}) = \begin{cases} c_i & \text{if } \mathbf{c} \in \mathcal{C} \text{ satisfies } d_H(\mathbf{c}, \mathbf{v}) \leq t; \\ v_i & \text{if } d_H(\mathbf{c}, \mathbf{v}) > t \text{ for all } \mathbf{c} \in \mathcal{C} \end{cases} \tag{1}$$

In words, the above decoder finds a codeword $\mathbf{c}$ whose Hamming distance to the received vector $\mathbf{v}$ is less than t, or if no such codeword can be found, it returns the received vector.

As for the overall code on the graph, normal message passing is used. The interesting point about their message passing algorithm is that the messages are binary, i.e. there are the received vectors/decoded codewords. This simplifies implementation A LOT!

However, the proposed decoding scheme is a bit inappropriate for neural implementation for a couple of reasons:

1. The constraint node should transmit different messages to each of its neighbors.
2. We have to apply the decoder D_i several times (n times) to decode each variable node since each time we have to take into account the channel output r_i in the decoding process,
3. Each variable node has to send two different values to the constraint nodes: the received channel value r_i and the message computed based on the last iterations messages $\mu_{i \to j'}^{(\ell+1)}$ such that the constraint node could decide the value of each variable node (albeit this property can be taken care of in a neural network).

In Lemma 11, the authors show that for a BSC channel with error probability $p^* = 2t/(2^\nu - 1)$, a spatially-coupled GLPDC code with rate $R = 1 - 2\nu t/(2^\nu - 1)$ achieves the ϵ-redundancy capacity. That is, for any $\epsilon > 0$, there exists a $V \in \mathbb{Z}_+$ such that for all $\nu \geq V$, we have

$$\frac{1 - C(p^*)}{1 - R} \geq 1 - \epsilon$$

3 Conclusions

Overall, the structure of the code proposed in [1] is very similar to our neural error correcting approach. And as mentioned in the previous report, there is a very nice method to analyze these codes proposed by Yedla et al. [2]. We will therefore work to apply this analysis to out neural graph and analytically inverstigate the performance of the algorithm.

References

[1] Y. Jian, H. D. Pster, K. R. Narayanan *Approaching capacity at high-rates with iterative hard-decision decoding*, Proc. IEEE Int. Symp. Inf. Theory (ISIT), 2012.

[2] A. Yedla, Y. Y. Jian, P. S. Nguyen, H. D. Pfister , *A simple proof of threshold saturation for coupled scalar recursions*, Proc. ISTC 2012.