~~COMPRESSION FOR THE AWGN CHANNEL~~

$$G : \begin{bmatrix} & \end{bmatrix}_{k \times n} \qquad n < k$$

~~Choose each column of G at random from set of all binary vectors of weight d. What is the prob that rank$(G) = n$ as $n, k \to \infty$?~~

SASON - WIECHMAN

$$H = \begin{bmatrix} & \end{bmatrix}_{(n-k) \times n}$$

Need the fraction of rows of weight k, denoted. Consider the Kolchin ensemble we used for the d ones are distributed uniformly at random among $(n-k)$ bins in each column. What is the prob that an number of ones landed in the first bin?

$$P(\text{odd \# in } 1^{st} \text{ bin}) = \left(\frac{1}{n-k}\right)\left(\frac{n-k-1}{n-k}\right)^{d-1} + \left(\frac{1}{n-k}\right)^3\left(\frac{n-k-1}{n-k}\right.$$

$$+ \cdots + \left(\frac{1}{n-k}\right)^d \quad \{d \text{ odd}\}$$

(OR)

$$\left(\frac{1}{n-k}\right)^{d-1}\left(\frac{n-k-1}{n-k}\right) \quad \{$$

Geometric progression $\dfrac{a(1-r^n)}{1-r}$

$$a = \left(\frac{1}{n-k}\right)\left(\frac{n-k-1}{n-k}\right)^{d-1} \qquad r = \left(\frac{1}{\cancel{n-k}}\right)^2 \Big/ \left(\frac{n-k-1}{\cancel{n-k}}\right)$$

$$n = \left\lceil \frac{d}{2} \right\rceil \qquad = \frac{1}{(n-k-1)^2}$$

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$2 \to 2$
$3 \to 4$
$4 \to 4$

P(odd # in 1^{st} bin)

$$= \left(\frac{1}{n-k}\right)\left(\frac{n-k-1}{n-k}\right)^{d-1} \frac{\left(1 - \frac{1}{(n-k-1)^{2\lceil d/2 \rceil}}\right)}{(n-k-1)^2 - 1} \cdot (n-k-1)^2$$

$$= \frac{(n-k-1)^{d+1-2\lceil d/2 \rceil}}{(n-k)^d} \cdot \frac{(n-k-1)^{2\lceil d/2 \rceil} - 1}{(n-k-1)^2 - 1}$$

$$\triangleq P_1$$

$$P(\text{row of weight } j) = \binom{n}{j} P_1^j (1-P_1)^{n-j} = \Gamma_j$$

$P_1 \sim O\left(\frac{1}{n}\right)$ { Intuitively, as $n \to \infty$, most entries will have zero 1s, which is an even number. Hence we should expect $P_1 \to 0$ as $n \to \infty$

$$\Gamma(x) = \sum_j \Gamma_j x^j$$

$$= \sum_{j=1}^{n} \binom{n}{j} P_1^j (1-P_1)^{n-j} x^j$$

$$= [1 - P_1 + P_1 x]^n$$

{ check: $\Gamma(1) = \sum_j \Gamma_j = 1$ ✓

$$\frac{H(X|Y)}{n} \geq 1 - C - (1-R)\left(1 - \frac{1}{2\ln 2}\sum_{p=1}^{\infty} \frac{\Gamma(g_p)}{p(2p-1)}\right)$$

As $n \to \infty$, we have that $\Gamma(x) > 0$ iff

$$\cancel{1} - P_1 + P_1 x \geq \cancel{1}$$

$$\Leftrightarrow x \geq 1$$

$$g_p \triangleq \int_0^{\infty} \underbrace{a(l)}_{\text{cond pdf of LLR given input zero}} (1+e^{-l}) \tanh^{2p}\left(\frac{l}{2}\right) dl$$

$\Gamma_{12} = \frac{1}{12}\left[\int_0 x^{11}\,dx\right] = \frac{1}{12}\cdot\frac{1}{\frac{[x^{12}]_0}{12}}$

BIAWGN :

$$a(\ell) = \sqrt{\frac{\sigma^2}{8\pi}}\; e^{-\left[\frac{\left(\ell-\frac{2}{\sigma^2}\right)^2\sigma^{-2}}{8}\right]}$$

(From Richardson, Shokrollahi & Urb Example 8

$$g_p = \int_0^\infty a(\ell)\tanh^{2p}\left(\frac{\ell}{2}\right)d\ell + \int_0^\infty a(\ell)\,e^{-\ell}\tanh^{2p}\left(\frac{\ell}{2}\right)d\ell$$

$\ell' = -\ell$

$$\int_0^{-\infty} a(-\ell')\,e^{\ell'}\tanh^{2p}\left(\frac{-\ell'}{2}\right)\,($$

$$= \int_{-\infty}^{0} a(\ell)\tanh^{2p}\left(\frac{\ell}{2}\right)d\ell$$

(from symmetry p

$$g_p = \int_{-\infty}^{\infty} a(\ell)\tanh^{2p}\left(\frac{\ell}{2}\right)d\ell$$ (for symmetric distr

$$= \int_{-\infty}^{\infty}\sqrt{\frac{\sigma^2}{8\pi}}\;e^{-\left[\frac{\left(\ell-\frac{2}{\sigma^2}\right)^2\sigma^{-2}}{8}\right]}\left[\frac{e^{\ell}-1}{e^{\ell}+1}\right]^{2p}d\ell \quad \left\{\because \tanh x = \frac{e^{2x}-1}{e^{2x}+1}\right.$$

$$g_p = \int_0^\infty a(\ell)\left(1+e^{-\ell}\right)\left[\tanh\left(\frac{\ell}{2}\right)\right]^{2p}d\ell$$

$$\leq \int_0^\infty a(\ell)\left(1+e^{-\ell}\right)d\ell = 1$$

$\{\because \tanh x \leq 1\}$

Hence we have that

$$\frac{H(X|Y)}{n} \geq \cancel{x} - C - (\cancel{x} - R) = R - C$$

$$I(X;Y) = H(X) - H(X|Y)$$
$$\leq \cancel{nR} - n(R-C)$$
$$= nC$$

As far as this bound is concerned, the LDPC erase seems optimal.