12/05/10

ANALYSING THE AWGN CHANNELS USING CBSCs

For a CBSC($\underline{P}$, $\underline{\lambda}$),

$$\rho = \sum_{i=1}^{N} \lambda_i (1 - 2p_i)^2$$

For the AWGN, $N = \infty$

pdf of the LLRs

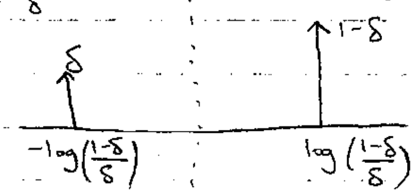
$$P_{AWGN}(y) = \sqrt{\frac{\sigma^2}{8\pi}}\; e^{-\frac{(y - \frac{2}{\sigma^2})^2 \sigma^2}{8}}$$

pdf of the LLRs for the BSC(δ)

$$P_{BSC}(y) = \delta\, \Delta_{-\log\frac{1-\delta}{\delta}} + (1-\delta)\, \Delta_{\log\frac{1-\delta}{\delta}}$$

Assume $\delta < 0.5$:

(i.e, $1-\delta > 0.5 > \delta$
$\Rightarrow \frac{1-\delta}{\delta} > 1$)



$$y = \log\left(\frac{1-\delta}{\delta}\right)$$

$$\frac{1-\delta}{\delta} = e^y \quad \Rightarrow \quad 1 - \delta = \delta e^y$$

$$\delta\,(e^y + 1) = 1$$

$$\delta = \frac{1}{1+e^y}$$

$$1 - \delta = \frac{\cancel{1} + e^y \cancel{-1}}{1+e^y} = \frac{e^y}{1+e^y}$$

amplitude of δ f
on the
half pl

$\therefore$, need to scale BSC pdf by

$$\frac{1+e^y}{e^y} \cdot \sqrt{\frac{\sigma^2}{8\pi}}\; e^{-(y - 2/\sigma^2)^2 \sigma^2/8}$$

To get a Δ function at y, need to have a crossover probability of $\delta = \frac{1}{1+e^y}$

$(\frac{1}{2})^{-\infty} = 2^{\infty} = \infty$ $\qquad$ $\rho = \frac{1}{\sigma^2} \Rightarrow \sigma = \frac{1}{\sqrt{\rho}}$

$.5(1-.2)^2 + .5(1-.4)^2 = .5[.64 + .36] = 1$

AWGN LLR

$$\phi_{AWGN} = \int_0^\infty \left(\frac{1+e^y}{e^y}\right)\sqrt{\frac{\sigma^2}{8\pi}}\, e^{-(y-2/\sigma^2)^2 \sigma^2/8} \cdot \phi_{\frac{1}{1+e^y}}$$

$$\rho_{AWGN} = \int_0^\infty \left(\frac{1+e^y}{e^y}\right)\sqrt{\frac{\sigma^2}{8\pi}}\, e^{-(y-2/\sigma^2)^2 \sigma^2/8} \left(1-\frac{2}{1+e^y}\right)$$

$$= \sqrt{\frac{\sigma^2}{8\pi}} \int_0^\infty \frac{e^y-1}{e^y}\, e^{-(y-\frac{2}{\sigma^2})^2 \sigma^2/8}\, dy$$

CBSC THRESHOLD SIMULATION:

$$u(\phi) = \left(\frac{1+e^{-2\phi d}}{2}\right)^{(1-\frac{1}{\phi})} \quad ; \quad u(0) = (\tfrac{1}{2})^{-\infty} = 2^{\infty}$$

As $\phi \to \infty$, $u(\phi) \to \left(\frac{1+0}{2}\right)^1 = 1/2$

Need to find the max ϕ s.t. $u(\phi) > \rho$

~~ρ will be less than 1/2 if all BSC transition probs are lesser than 1/2.~~

For the AWGN approx case, transition probs $\frac{1}{1+e^y}$ are all $\leq 1/2$

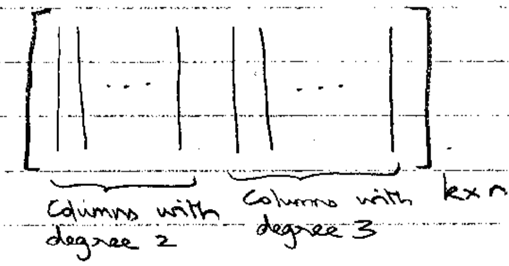
If $\rho \leq 1/2$, the maximizing ϕ will be ∞

For $\rho > 1/2$, need to simulate.

$u(1) = 1$, this is always $\geq$ any $\rho \Rightarrow$ can start searching for all $\phi \geq 1$

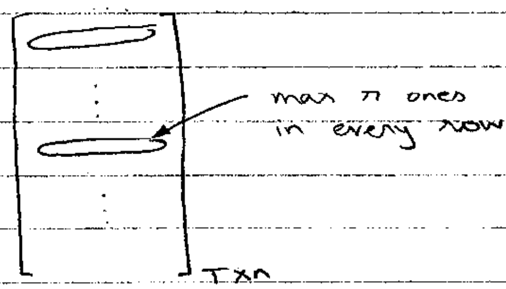
14/05/10

GENERATOR MATRICES WITH SEVERAL $^{O/P}$ DEGREES



Suppose that k is fixed. Can have $\theta(2,\tau)k$ columns of deg 2, with no MI loss. Then say that we have $\theta(3,\tau)k$ cols. of degree 3. What is the value of $\theta(3,\tau)$ such that we are at capacity?

$A_{T\times n} =$



(Kolchin)

$$P(\xi_{t_1,t_2,\ldots,t_m} = 1) = P(\text{rows } t_1, t_2, \ldots, t_m \text{ sum to } \underline{0})$$

$$= P_E(rm, n).$$

$$= P(\text{vector of weight } m \text{ lying in the left null-space of } A)$$

Compute: $\sum_{m=1}^{T} \binom{T}{m} P_E(rm, n)$

<u>BSC</u>:

$$\sum_{w=1}^{n} B_w (1-2p)^{2w}$$

B_w: # of n-tuples of weight w in the right null-sp. of $G_{k\times n}$ = # of n-tuples of weight w in left null-sp. of $(G^T)_{n\times k}$

$\therefore\ (G^T)_{n\times k} \equiv A_{T\times n}$

Suppose that $A_{T\times n}$ has the first $\theta_1 T$ rows of weight 2, and the next $\theta_2 T$ rows of weight 3 such that $\theta_1 + \theta_2 = 1$.
Then $P(\xi_{t_1,\dots,t_m} = 1)$ will depend on how many of the $t_1,\dots,t_m$ are chosen from the first $\theta_1 T$

Possible splits:

First $\theta_1 T$ rows	Next $\theta_2 T$ rows	# of such m-tuples	P(such an m-tuple being in the
1	m−1	$\binom{\theta_1 T}{1}\cdot\binom{\theta_2 T}{m-1}$	$P_E(1\cdot 2 + (m-1)\cdot 3,$ n
2	m−2	$\binom{\theta_1 T}{2}\cdot\binom{\theta_2 T}{m-2}$	$P_E(2\cdot 2 + (m-2)\cdot 3,$
⋮		⋮	⋮
m−1	1	$\binom{\theta_1 T}{m-1}\cdot\binom{\theta_2 T}{1}$	$P_E((m-1)2 + 1\cdot 3,$
m	0	$\binom{\theta_1 T}{m}\cdot 1$	$P_E(2m, n)$
0	m	$1\cdot\binom{\theta_2 T}{m}$	$P_E(3m, n)$

∴ We evaluate

$$\sum_{m=1}^{T}\left[\sum_{i=0}^{m}\binom{\theta_1 T}{i}\binom{\theta_2 T}{m-i} P_E\left(i(2)+(m-i)3, n\right)\right]$$

$$\leq \sum_{m=1}^{T}(\cosh\lambda)^n\sum_{i=0}^{m}\binom{\theta_1 T}{i}\binom{\theta_2 T}{m-i}\frac{[i(2)+(m-i)(3)]!}{(\lambda n)^{2i+3(m-i)}}$$

17/05/10

RANDOM THOUGHTS:

- Check to see if there are results extending threshold behaviour for binary systems to GF Discussion following (8) in Levitskaya's paper seem to suggest that this is possible.
But in order to use these results, we need to get bounds on MI for non-binary LDPC/raptor codes – think about these.

$$\frac{T(T-1)\cdots(T-(m-1))}{\cancel{T!}\; m!\,\cancel{(T-m)!}}$$

ANALYZING THE MULTIPLE DEGREE CASE:

Extension of Lemma 3.5.1 of Kolchin:

Let $T_1 = \theta_1 T$, $T_2 = \theta_2 T$.

$$S_1 = \sum_{1 \le m \le \delta T} \left[\sum_{i=0}^{m} \binom{T_1}{i} \binom{T_2}{m-i} P_E(2i + 3(m-i), n) \right]$$

$$\le \sum_{1 \le m \le \delta T} \sum_{i=0}^{m} \frac{T_1^i}{i!} \cdot \frac{T_2^{m-i}}{(m-i)!} (\cosh \lambda_i)^n \frac{(2i+3(m-i))!}{(\lambda n)^{2i+3(m-i)}}$$

Put $x_i = \dfrac{2i+3(m-i)}{n} = \lambda_i \tanh \lambda_i$, $i = 1, \dots, m-1$

$$\cancel{S_1 \le \sum_{1 \le m \le \delta T} \sum_{i=1}^{m-1}}$$

$$\frac{(2i+3(m-i))!}{i!\,(m-i)!} = \frac{(i+1)(i+2)\cdots(2i+3(m-i))}{(m-i)!} \le \frac{(2i+3(m-i))^{i+3(m-i)}}{(m-i)!}$$

$$\le (2i+3(m-i))^{i+3(m-i)}$$

$$\cosh \lambda_i \le e^{4x_i}$$

$$S_1 \le \sum_{1 \le m \le \delta} \sum_{i=0}^{m} T_1^i \, T_2^{m-i} \, e^{4x_i n} \frac{(2i+3(m-i))^{i+3(m-i)}}{(\lambda n)^{2i+3(m-i)}}$$

$$\le \sum_{1 \le m \le \delta T} \sum_{i=0}^{m} \left(\frac{T_1}{n}\right)^i \left(\frac{T_2}{n}\right)^{m-i} \cdot e^{4[2i+3(m-i)]} \frac{(2i+3(m-i))^{i+3(m-i)}}{(\sqrt{x_i})^{2i+3(m-i)}\; n^{i+2(m-i)}}$$

$$= \;\; '' \;\; '' \;\; '' \;\; '' \;\; '' \cdot \frac{(2i+3(m-i))^{i+3(m-i)}}{\left[\frac{(2i+3(m-i))}{n}\right]^{\frac{2i+3(m-i)}{2}} \cdot n^{i+}}$$

$$\cancel{i}+3(m-i)\cancel{-i}-\tfrac{3}{2}(m-$$

$$= \quad '' \quad \cdots \cdot \frac{\left[m\left(3-\frac{3i}{m}+\frac{2i}{m}\right)\right]}{n^{i+2(m-i)-1}}$$

$$= \quad '' \quad \cdots \cdot \frac{\left(\frac{m}{n}\right)^{\frac{3}{2}(m-i)} \cdot \left(3-\frac{3i}{m}+\frac{2i}{m}\right)^{\frac{3}{2}(m-i)}}{n^{i+\frac{(m-i)}{2}-1}}$$

$$\leq \quad '' \quad \cdots \cdot \left(\frac{\alpha m}{T}\right)^{\frac{3}{2}(m-i)} \left(3-\frac{3i}{m}+\frac{2i}{m}\right)^{\frac{3}{2}(m-i)}$$

$$= \sum_{1\leq m\leq \delta T} \sum_{i=0}^{m} \left[\underbrace{\left(\frac{T_1}{n}\right)^{i/m}}_{\to\text{const}} \underbrace{\left(\frac{T_2}{n}\right)^{1-i/m}}_{\to\text{const.}} \overbrace{e^{4\left[2i/m+3(1-i/m)\right]}}^{\text{const. (wrt: } T, n)} \cdot \left(\frac{\alpha m}{T}\right)^{\frac{3}{2}(1-\cdots} \underbrace{\left(3-\frac{3i}{m}+\frac{2i}{m}\right)^{\frac{3}{2}(1-i/m)}}_{\text{const.}} \right]^m$$

$\frac{m}{T} \leq \delta \Rightarrow S_1$ can be made arbitrarily sm[...] by choosing δ small.

Lemma 3.5.2:

$$S_2 = \sum_{(1-\delta)T\leq m\leq T} \left[\sum_{i=0}^{m} \binom{T_1}{i}\binom{T_2}{m-i} P_E\left(2i+3(m-i), n\right) \right]$$

Let $\lambda = \frac{2i+3(m-i)}{n}$ and $m_0 \in \mathbb{Z}$ s.t. $\frac{m_0}{T} \leq$ [...]

$$S_2 \leq \sum_{m=T-m_0}^{T} \left[\sum_{i=0}^{m} \binom{T_1}{i}\binom{T_2}{m-i} (\cosh\lambda)^n \underbrace{\frac{(2i+3(m-i))!}{(2i+3(m-i))^{2i+3(m-i)}}} \right]$$

$$\leq \frac{c\,(\cancel{2i+3(m-i)})^{\cancel{2i+3(m-i)}}\, e^{-(2i+3(m\cdots} \; (2i+3(T-i))}{(2i+3(\cancel{m-i}))^{2i+3(m-i}}$$

$m \geq T - m_0$
$T - m \leq m_0$

$$S_2 \leq c \sum_{m=T-m_0}^{T} \left[\sum_{i=0}^{m} \binom{T_1}{i}\binom{T_2}{m-i} \left(e^{-\lambda}\cosh\lambda\right)^n \left(2i+3(m-i)\right)^{1/2} \right]$$

$$e^{-\lambda}\cosh\lambda = \frac{(1+e^{-2\lambda})}{2} \leq q \quad , \quad \text{some } q<1 .$$

$$S_2 \leq c q^n \sum_{m=T-m_0}^{T} \sum_{i=0}^{m} \binom{T_1}{i}\binom{T_2}{m-i} \underbrace{(2i+3(m-i))^{1/2}}_{\text{decreasing fn of } i}$$

$$\leq c q^n \sum_{m=T-m_0}^{T} (2+3(m-1))^{1/2} \underbrace{\sum_{i=0}^{m} \binom{T_1}{i}\binom{T_2}{m-i}}$$

A&S, pg. 822, 24.II.B:

$$\sum_{m=0}^{n} \binom{r}{m}\binom{s}{n-m} = \binom{r+s}{n} \quad , \quad r+s \geq n$$

$m \leftarrow i$, $n \leftarrow m$, $r \leftarrow T_1$, $s \leftarrow T_2$:

$$\sum_{i=0}^{m} \binom{T_1}{i}\binom{T_2}{m-i} = \binom{T_1+T_2}{m}$$

$$S_2 \leq c q^n \sum_{m=T-m_0}^{T} \binom{T}{m} (2+3(m-1))^{1/2}$$

$$\leq c q^n (3T-1)^{1/2} \sum_{m=T-m_0}^{T} \binom{T}{m}$$

Can now proceed exactly as in the original case.

Theorem 3.5.1 :

$\cancel{x_i = \frac{2i+3(m-i)}{n}, \quad i = 1, \dots, m-1}$

$\cancel{T, n \to \infty, \quad T/n \to \alpha}$

$\cancel{P_E(2i+3(m-i), n) = (\cosh\lambda)^n \left(\frac{x}{\lambda e}\right)^{xn} \frac{2\sqrt{x}}{\sigma} (1+ \quad \sigma^2 = \lambda^2 + x}$

Approach 1 : $\binom{a}{b} \leq \left(\frac{ae}{b}\right)^b$

$$P_E(2i+3(m-i), n) \leq (\cosh\lambda)^n \frac{(2i+3(m-i))!}{(\lambda n)^{2i+3(m-i)}}$$

$$T_i = \binom{T_1}{i}\binom{T_2}{m-i} P_E(2i+3(m-i), n)$$

$$\leq \left(\frac{T_1 e}{i}\right)^i \left(\frac{T_2 e}{m-i}\right)^{m-i} (\cosh\lambda)^n \frac{(2i+3(m-i))!}{(\lambda n)^{2i+3(m-i)}}$$

$$\leq \frac{e^m \theta_1^i \theta_2^{m-i} T^m}{i^i (m-i)^{m-i}} (\cosh\lambda)^n \cdot \frac{2[2i+3(m-i)]\left[\frac{2i+3(m-i)}{e}\right]^{2i+3(m-i)}}{(\lambda n)^{2i+3(m-i)}}$$

$$\left\{ \because n! \leq 2n \left(\frac{n}{e}\right)^n \right\}$$

$$= \left[\underbrace{(Te)^{m/T} \left(\frac{\theta_1}{i}\right)^{i/T} \left(\frac{\theta_2}{m-i}\right)^{\frac{m-i}{T}}}_{\leq \left(\frac{Te}{m}\right)^{m/T} \theta_1^{i/T} \theta_2^{\frac{m-i}{T}}} \cdot (\cosh\lambda)^{n/T} \cdot [2(2i+3(m-i))]^{1/T} \cdot \left[\frac{2i+3(m-i)}{\lambda n e}\right]^{\frac{2i+3(m-i)}{T}} \right.$$

$$i^i (m-i)^{m-i} \leq m^i (m)^{m-i} = m^m$$

$\cancel{\text{Let } \alpha = \frac{m}{n}, \quad \phi = \frac{T}{n}}$.

$\cancel{\sqrt[T]{\theta_1^i} \to 1, \quad \sqrt[T]{\theta_2^{-i}} \to 1 \text{ as } T \to \infty}$

$\cancel{\sqrt[T]{2(2i+3(m-i))} \to 1 \text{ as } T \to \infty}$ $\boxed{\text{CHECK!}}$

$$\frac{d}{dx}\left[x^x\right] = x \cdot x^{x-1} \quad (a+b)^2 = a^2+b^2+2ab \quad (a+b)^2 \le a^2+b^2$$

$$(a+b+c)^2 \quad \sqrt{a+b} \quad \sqrt{a}+\sqrt{b}$$

$\theta_1 > \theta_2 : 1$
$\theta_1 < \theta_2 : \left(\frac{\theta_1}{\theta_2}\right)^m$

~~Need to ensure that:~~

$$\cancel{\frac{x}{\phi} \ln\left(\frac{\phi e}{x}\right)} + \frac{x}{\phi} \ln \theta_2 + \cancel{\frac{1}{\phi} \ln} \cosh \lambda$$

$$\sum_{i=0}^{m} T_i \le \left(\frac{Te}{m}\right)^m \theta_2^m \, (\cosh\lambda)^n \, (2)$$

$$\sum_{i=0}^{m} (3m-i) \left(\frac{3m-i}{\lambda n e}\right)^{3m-i} \left(\frac{\theta_1}{\theta_2}\right)^i$$

$$\le \left(\frac{Te}{m}\right)^m \theta_2^m \, (\cosh\lambda)^n \, (2) \, . \, m \, (3m) \left(\frac{3m}{\lambda n e}\right)^{3m}$$

$$f'(\theta_1, \theta_2),$$

$$\text{where } f'(\theta_1,\theta_2) = \begin{cases} 1 & \text{if } \theta_1 > \theta_2 \\ \left(\frac{\theta_1}{\theta_2}\right)^m & \text{if } \theta_1 < \theta_2 \end{cases}$$

$$= 6m^2 \left(\frac{Te}{m}\right)^m \left[\min(\theta_1,\theta_2)\right]^m (\cosh\lambda)^n \left(\frac{3m}{\lambda n e}\right)^{3m}$$

$$= 6m^2 \left[\frac{Te}{m} \min(\theta_1,\theta_2) \cdot \left(\frac{3m}{\lambda n e}\right)^3\right]^m (\cosh\lambda)^n$$

Set $x = \frac{m}{n}$, $\phi = T/n$

$$\sum_{i=0}^{m} T_i \le \Bigg[\underbrace{6^{1/T} \, (m^2)^{1/T}}_{\to 1 \text{ as } T \to \infty} \left(\frac{Te}{m} \min(\theta_1,\theta_2) \cdot \left(\frac{3m}{\lambda n e}\right)^3\right)^{m/T} (\cosh\lambda)^{n/T}$$

Need to ensure that

$$\frac{x}{\cancel{\phi}} \ln\left(\frac{\phi e}{x} \cdot \min(\theta_1,\theta_2) \cdot \left(\frac{3x}{\lambda e}\right)^3\right) + \frac{1}{\cancel{\phi}} \ln \cosh\lambda < 0$$

$$\Leftrightarrow \left[\frac{\phi e}{x} \min(\theta_1,\theta_2) \left(\frac{3x}{\lambda e}\right)^3\right]^x \cdot \cosh\lambda < 1$$

Set $x = \frac{\lambda \tanh \lambda}{d}$, where $d = 2\theta_1 + 3\theta_2$

$$\left[\frac{\phi e d}{\lambda \tanh \lambda} \min(\theta_1, \theta_2) \left(\frac{3 \cancel{\lambda} \tanh \lambda}{d \cancel{\lambda} e} \right)^3 \right]^{\frac{\lambda \tanh \lambda}{d}} \cosh \lambda <$$

$$\frac{\phi e d}{\lambda \tanh \lambda} \min(\theta_1, \theta_2) \left(\frac{3 \tanh \lambda}{d e} \right)^3 (\cosh \lambda)^{d/\lambda \tanh \lambda} <$$

$$\phi < \frac{\lambda \tanh \lambda}{e d \min(\theta_1, \theta_2)} \left(\frac{d e}{3 \tanh \lambda} \right)^3 (\cosh \lambda)^{-d/\lambda \tanh \lambda}$$

$$\triangleq f(\lambda)$$

{ Need to include the $(1-2p)^{2m}$ for the BSC channel }

Approach 2 :

$$\binom{T}{m} = \frac{\phi}{\sqrt{2\pi x(\phi - x)\phi k}} \left(\frac{\phi^\phi (\phi - x)^x}{x^x (\phi - x)^\phi} \right)^k (1 + o(1)),$$

where

$$x = \frac{m}{n}, \quad \phi = T/n$$

$$\binom{T}{m} = \frac{1}{p^m q^{T-m} \sqrt{2\pi T p q}} (1 + o(1)), \quad p = \frac{m}{T}, \quad q =$$

$$\binom{T_1}{i} \binom{T_2}{m-i} = \frac{1}{\left(\frac{i}{T_1}\right)^i \left(1 - \frac{i}{T_1}\right)^{T_1 - i} \sqrt{2\pi \cancel{T_1} \left(\frac{i}{\cancel{T_1}}\right)\left(1 - \frac{i}{T_1}\right)}} \cdot$$

$$\frac{1}{\left(\frac{m-i}{T_2}\right)^{m-i} \left(1 - \frac{m-i}{T_2}\right)^{T_2 - m + i} \sqrt{2\pi \cancel{T_2} \left(\frac{m-}{\cancel{T_2}}\right.}}$$

$$= \frac{T_1^{T_1} \sqrt{T_1}}{i^i (T_1 - i)^{T_1 - i} \sqrt{2\pi i (T_1 - i)}} \cdot \frac{T_2^{T_2} \sqrt{T_2}}{(m-i)^{m-i} (T_2 - m + i)^{T_2 - m + i} \sqrt{2\pi (m-i)(T_2 - m + i)}}$$

For all $1 \leq i \leq m-1$,

$$\binom{T_1}{i}\binom{T_2}{m-i} \leq \frac{\sqrt{T_1T_2}\; T_1^{T_1}\; T_2^{T_2}}{2\pi (1)^1 \left(T_1-(m-1)\right)^{T_1-m+1}\left(T_2-m+1\right)^{T_2-m+1}\sqrt{(T_1-m+1)(T_2-m\cdots}}$$

$$= \frac{T\sqrt{\theta_1\theta_2}\;(T\theta_1)^{T\theta_1}\;(T\theta_2)^{T\theta_2}}{2\pi\;(T\theta_1-m+1)^{T\theta_1-m+3/2}\,(T\theta_2-m+1)^{T\theta_2-m+3/2}}$$

$$= \frac{T^{(T+1)}\left(\theta_1^{\theta_1}\theta_2^{\theta_2}\right)^T\sqrt{\theta_1\theta_2}}{2\pi\;(T\theta_1-m+1)^{T\theta_1-m+3/2}\,(T\theta_2-m+1)^{T\theta_2-m+3/2}}$$

$$= \left[\frac{T^{(1+1/T)}\;\theta_1^{\theta_1}\;\theta_2^{\theta_2}\;\left(\sqrt{\theta_1\theta_2}\right)^{1/T}}{(2\pi)^{1/T}\;(T\theta_1-m+1)^{\theta_1-m/T+3/2T}\;(T\theta_2-m+1)^{\theta_2-m/T+3/2T}}\right]^T$$

$$T_i \leq m\left[\qquad \text{''} \qquad\right]^T \cdot \left[(\cosh\lambda)^{n/T}\cdot\left[2(2i+3(m-i))\right]^{1/T}\cdot\underbrace{\left(\frac{2i+3(m-i)}{\lambda n e}\right)^{\frac{2i+3(m-i)}{T}}}_{\text{set } i=1 \text{ to maximize}}\right]$$

Set $x = \frac{m}{n}$, $\phi = \frac{T}{n}$.

$$T_i \leq \left[\left(\frac{m\sqrt{\theta_1\theta_2}}{2\pi}\right)^{1/T}\cdot\frac{T^{1+1/T}\;(\cosh\lambda)^{n/T}\;[2(3m-1)]^{1/T}\;\left(\frac{3m-1}{\lambda n e}\right)^{\frac{3m-1}{T}}}{T^{1-2m/T+3/2T}\left(\theta_1-\frac{m}{T}+\frac{1}{T}\right)^{\theta_1-m/T+3/2T}\left(\theta_2-m/T+1/T\right)^{\theta_2-m/T+3/2T}}\right]$$

7/5/10 MIKHAILOV'S PAPER (Vol 41, No. 2, THEORY PROBAB. APP

$$\gamma_\phi(z) = d - 2^d\sum_{k=0}^{d-1}\frac{z^k(1-z)^{d-1-k}}{k!\,(d-1-k)!}\,2^d, \quad 0 \leq z \leq 1$$

$$= d - 2^{2d}\sum_{k=0}^{d-1}\frac{z^k}{k!}\cdot\frac{(1-z)^{d-1-k}}{(d-1-k)!}$$

$$\triangleq \gamma_d'(z)$$

$$[\,G\,]_{k\times n}\,[\;] = \underline{0}$$

k eqns, n vars.

Suppose that all eqns are of the same degree d.
Then

$$n \cdot \gamma_d'(z) = N(\log N + z), \quad z = O(1)$$

(n eqns., N variables)

The analysis in this paper will not work to our case, since:

(1) the paper doesn't consider a homogeneous & of equations

(2) The number of eqns n and variables N scale

$$n = N \log N$$

HOMOGENEOUS/NON-HOMOGENEOUS:

$$A\underline{x} = \underline{b}, \quad \underline{b} \neq \underline{0}$$

Suppose $\exists\ \underline{x} = \underline{x}_b$, such that $A\underline{x}_b = \underline{b}$

$$A\underline{x} - \underline{b} = \underline{0}$$

$$A(\underline{x} - \underline{x}_b) = \underline{0}$$

31/05/10

MUTUAL INFO: MULTIPLE COLUMN DEGREES:

$$\Big[\underbrace{\,|\,|\,\cdots|\,|\,}_{\theta_1 n \triangleq n_1}\underbrace{|\,\cdots\,|}_{\theta_2 n \triangleq n_2}\Big]_{k\times n} = G = [\underset{k\times n_1}{G_1}\ \ \underset{k\times n_2}{G_2}]$$

$$\underline{y} = G^T\underline{x} + \underline{w}, \quad \underline{w} \sim \text{iid } N(0, \tfrac{1}{\rho} I)$$

$$= \begin{bmatrix} G_1^T \\ G_2^T \end{bmatrix}\underline{x} + \underline{w} = \begin{bmatrix} G_1^T\underline{x} \\ G_2^T\underline{x} \end{bmatrix} + \underline{w} \triangleq \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix}$$

$$H(\underline{y}) = H(\underline{y}_1; \underline{y}_2) = \underbrace{H(\underline{y}_1)}_{\text{can analyze using existing techniques}} + H(\underline{y}_2 \mid \underline{y}_1)$$

$$H(\underline{y}_2 \mid \underline{y}_1) = ?$$

$$H(G_2^T \underline{x} + \underline{w}_2 \mid G_1^T \underline{x} + \underline{w}_1)$$

Gaussian case:

When $\underline{x}(u)$ & $\underline{y}(u)$ are jointly Gaussian,

$$P_{\underline{y}(u)|\underline{x}(u)}(\underline{w} \mid \underline{v}) = N_\wedge\left(\underline{w} - [\underline{m}_y + K_{yx}K_x^{-1}(\underline{v} - \underline{m}_x)],\ K_y - K_{yx}K_x^{-1}K_{xy}\right.$$

Given that $\underline{y} = \underline{y}^{[i]} = \begin{bmatrix} \underline{y}_1^{[i]} \\ \underline{y}_2^{[i]} \end{bmatrix}$ was transmitted,

$\underline{m}_{y_1} = \underline{y}_1^{[i]}$, $\underline{m}_{y_2} = \underline{y}_2^{[i]}$. Let $\underline{y}^{[i]} = G^T \underline{x}^{[i]}$

$$K_{y_1 y_2} = \mathbb{E}\left[(\underline{y}_1 - \underline{m}_{y_1})(\underline{y}_2 - \underline{m}_{y_2})^+\right]$$

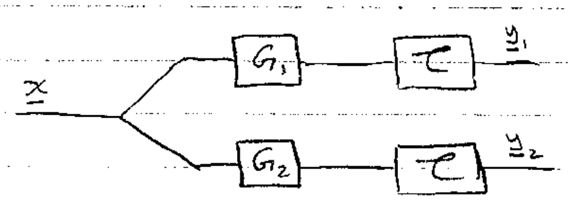
$$= \mathbb{E}\left[\underline{y}_1 \underline{y}_2^+ - \underline{m}_{y_1}\underline{y}_2^+ - \underline{y}_1 \underline{m}_{y_2}^+ + \underline{m}_{y_1}\underline{m}_{y_2}^+\right]$$

$$\mathbb{E}[\underline{y}_1 \underline{y}_2^+] = \mathbb{E}\left[(-1)^{G_1^T \underline{x}} \quad (-1)^{\underline{x}^T G_2}\right]$$

$$= \mathbb{E}\left[(2\underline{y}_1 - \underline{1})(2\underline{y}_2 - \underline{1})^+\right]$$

$$= \mathbb{E}\left[4\underbrace{\underline{y}_1 \underline{y}_2^+}_{= G_1^T \underline{x}\,\underline{x}^T G_2} - 2\underline{y}_1 \underline{1}^+ - 2\underline{1}\,\underline{y}_2^+ + \underline{1}\,\underline{1}^+\right]$$

How does one compute these covariances over GF(2) properly?



$a \quad b$ $\qquad ac+bd \qquad (a+b)(c+d)$
$c \quad d$

Let B_W and C_W denote the # of codewords of weight w, in the first $\theta_1 n$ code-bits & the l
$\theta_2 n$ code bits. Let D_W denote the resultant wei
distrib.

$$D_W = \sum_{i=0}^{w} B_i C_{w-i}$$

$$S = \sum_{w=0}^{n} D_W \rho^w = \sum_{w=0}^{n} \rho^w \sum_{i=0}^{w} B_i C_{w-i}$$

B_0	B_1	B_2	$\cdots$		$B_{\theta_1 n-1}$	$B_{\theta_1 n}$
C_0	C_1	C_2	$\cdots$	$C_{\theta_2 n-1}$	$C_{\theta_2 n}$	

C.S. inequality:

$$S \leq \sum_{w=0}^{n} \rho^w \sqrt{\sum_{i=0}^{w} B_i^2} \cdot \sqrt{\sum_{i=0}^{w} C_i^2}$$

$$S = \sum_{w=0}^{n} \sum_{i=0}^{w} \left[(B_i \rho^i)(C_{w-i} \rho^{w-i}) \right]$$

$$\leq \sum_{w=0}^{n} \left[\sum_{i=0}^{w} B_i \rho^i \right] \left[\sum_{i=0}^{w} C_{w-i} \rho^{w-i} \right] \quad , \text{ since } B_i, C_i$$

$$= \sum_{w=0}^{n} \left[\sum_{i=0}^{w} B_i \rho^i \right] \left[\sum_{i=0}^{w} C_i \rho^i \right]$$