$$W = \frac{2^{k-2}}{N} G^T \left\{ I_k + \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \end{bmatrix}_{k\times k} \right\} G_{k\times n}$$

$$= \frac{2^{k-2}}{N} G^T G + \frac{2^{k-2} d}{N} \underbrace{\underbrace{G^T}_{n\times k} \begin{bmatrix} 1 & \cdots & 1 \\ 1 & \cdots & 1 \\ 1 & \cdots & 1 \end{bmatrix}_{k\times n}}_{\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \end{bmatrix}_{n\times n}}$$

$$w_{ij} = \frac{2^{k-2}}{N} \left[\underline{g}_i^T \underline{g}_i + d \right]$$

$$W = \frac{1}{N} \left[4\, \Xi_2^T \Xi_2 - 2\, \Xi_2^T \underline{\underline{1}} - 2\, \underline{\underline{1}}^T \Xi_2 + \underline{\underline{1}}^T \underline{\underline{1}} \right]$$

Need to take care of real vs. GF_2 operations.

Codeword generation:

Define $\Xi = A G$ real ops.

Actual codewords $= \Xi \bmod 2$

Actual patterns $= 2\,\Xi_{\bmod 2} - \underline{\underline{1}} \triangleq 2\,\Xi_2 - 1$

$$\Xi_2^T \Xi_2 = (G^T A^T)_{\bmod 2} (AG)_{\bmod 2}$$

$$\neq \big(\underbrace{G^T A^T A G}_{\text{all real ops.}}\big)_{\bmod 2}$$

$$\cancel{= (G^T G)_{\bmod 2}}$$

$$\cancel{(A^T A)_{\bmod 2} = 0 \Rightarrow \Xi_2^T \Xi_2 = 0}$$

$$\underset{n\times 2^k}{\underline{\underline{1}}^T} \underset{2^k\times n}{\underline{\underline{1}}} = 2^k\, \underline{\underline{1}}_{n\times n}$$

$$\underset{n\times 2^k}{\Xi_2^T} \underset{2^k\times n}{\underline{\underline{1}}} = 2^{k-1}\, \underline{\underline{1}}_{n\times n} = \underline{\underline{1}}^T \Xi_2$$

$$W = \frac{1}{N}\left[4\, \Xi_2^T \Xi_2 - 2^{k+1} \underline{\underline{1}}_{n\times n} + 2^k \cdot \underline{\underline{1}}_{n\times n}\right]$$

$$= \frac{1}{N}\left[4\, \Xi_2^T \Xi_2 - 2^k \underline{\underline{1}}_{n\times n}\right]$$

Suppose that $W = \Gamma^T \Delta \Gamma$

$\downarrow$ should map vectors of small Euclid norm to vectors of small ∞-norm

Check to see: Sum a few cols of the Hadamard matrix, see what the obtained ∞-norm is

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<u>MAC</u> <u>DISTORTION</u> <u>EXPONENTS</u>:

Informed tx upper bound, a closer look:

Equivalent MIMO system

$$b' = \frac{T}{|K| S} = \frac{b}{|K|}$$

Rate of transmission $R' = |K|\, R(H)$

Distortion obtained for the MIMO equivalent

$$D'(P) = \mathbb{E}\left[e^{-b'R'}\right] \quad \left\{ \text{Source-channel separation is optimal here} \right.$$

$$= \mathbb{E}\left[e^{-b R(H)}\right]$$

Distortion for the actual system is atleast equal to of the MIMO system:

$$D(P) \geq D'(P) = \mathbb{E}\left[e^{-b R(H)}\right]$$

$$\geq \mathbb{E}\left[e^{-b \cdot \frac{1}{|K|} \log\det(\quad)}\right] \quad \forall\, |K|$$

MODIFYING THE HOPFIELD METRIC

$$W_{ij} = \frac{1}{N}\sum_{m=1}^{M} \xi_i^m \xi_j^m \equiv \frac{1}{N}\,\Xi^T\,\Xi \triangleq W$$

$$\downarrow$$

$$\frac{1}{N}\sum_{m=1}^{M} \Omega_m\, \xi_i^m \xi_j^m \equiv \frac{1}{N}\,\Xi^T \underbrace{\begin{bmatrix} \Omega_1 & & & \\ & \Omega_2 & & \\ & & \ddots & \\ & & & \Omega_M \end{bmatrix} \Xi}_{= \begin{bmatrix} | & & | \\ \xi^1 & \cdots & \xi^M \\ | & & | \end{bmatrix} \begin{bmatrix} \text{---}\ \Omega_1 \xi^1\ \text{---} \\ \text{---}\ \Omega_2 \xi^2\ \text{---} \\ \vdots \\ \text{---}\ \Omega_M \xi^M\ \text{---} \end{bmatrix}} \triangleq W$$

What about $\frac{1}{N}\Xi^T\Omega\,\Xi$, for Ω not necessarily diagonal

$$W_{ij} = \frac{1}{N}\sum_{k,l} (\Xi^T)_{ik}\,\Omega_{kl}\,\Xi_{lj}$$

Recovery equations:

$$x_i^{(t+1)} = \max\Big\{\sum_{j\neq i} W_{ij}\, x_j^{(t)},\ 0\Big\} = \max\{\underline{0},\ W_n x^{(t)}\}$$

~~Suppose that~~

Diagonal entries of W':

$$W_{ii} = \sum_{m=1}^{M}\Omega_m (\xi_i^m)^2 = \sum_{m=1}^{M}\Omega_m$$

$$W_n = W' - \Big(\sum_{m=1}^{M}\Omega_m\Big) I/N$$

Suppose we feed in a pattern ξ^1:

$$W_n\xi^1 = W'\xi^1 - \frac{1}{N}\Big(\sum_{m=1}^{M}\Omega_m\Big)\xi^1$$

Assume that the ξ^m are (almost) orthogonal

$$W_n\xi^1 = \frac{1}{\cancel{N}}\xi^1(\Omega_1 \cancel{N}) - \frac{1}{N}\Big(\sum_{m=1}^{M}\Omega_m\Big)\xi_1$$

{ Perfectly orthogonal, so $M=N$ in this case

$$= \xi^1\Big[\Omega_1 - \frac{\sum_{m=1}^{M}\Omega_m}{N}\Big]$$

$k = \frac{k}{n}$
$n = k/R$
$2^k < n$
$2^{Rn} < n$ $\quad Rn < \log_2 n$
$R < \frac{\log_2 n}{n}$

$\cancel{\quad} > \frac{\alpha M}{N}$

$\Omega_1 - \frac{\alpha M}{N}$
$N\Omega_1 > \alpha N$
$\Omega_1 > \frac{\alpha M}{N}$

$$W_n \xi' = \frac{1}{\cancel{N}} \xi'(\Omega_1 \cancel{N}) + \frac{1}{N}\xi^2 \Omega_2 <\xi^2, \xi'> + \frac{1}{N}\xi^3 \Omega_3 < \ldots + \frac{1}{N}\xi^M \Omega_M <\xi^M, \xi'> - \frac{\sum_{m=1}^{M} \Omega_m}{N} \xi_1$$

$$= \xi_1 \underbrace{\left[\Omega_1 - \frac{\sum_{m=1}^{M} \Omega_m}{N} \right]}_{\text{would like this to be positive (N large helps, if all } \Omega_i \text{ +ve)}} + \frac{1}{N} \underbrace{\sum_{m=2}^{M} \xi^m \Omega_m <\xi^m, \xi'}_{\text{would lik keep the m of all cor small}}$$

Observations:

① Suppose that $\sum_{m=1}^{M} \Omega_m \approx c$

Then $\Omega_1 - \frac{\sum_{m=1}^{M} \Omega_m}{N} > 0$

$\Leftrightarrow \quad \Omega_1 > \frac{c}{N} \quad \Rightarrow \sum_{m=1}^{M} \Omega_m > \frac{Mc}{N}$

$\cancel{c} > \frac{M\cancel{c}}{N}$

$\boxed{N > M}$ if $c > 0$

$\boxed{M > N}$ if $c < 0$

Gold sequences:

Length of the sequences $n = 2^m - 1$
# of sequences $= 2^m + 1$ (known these are all cyclically d
max $\{ |$corr. betn cyclic shifts of two Gold seqs.$| \} = 1 + \sqrt{2^{m+}}$
$= 1 + \sqrt{2(n+1)}$
All these correlations are with sequences with norm 1
(this is trivially satisfied for binary seqs, being $\xi^\pm$

# of seqs including cyclic shifts

$$= n\,(2^m + 1)$$

$$= n\,(n+2) = n^2 + 2n$$

What happens to the correlation term in this case?

$$\left| \frac{1}{N} \sum_{m=2}^{M} \xi_m \, \Omega_m \, \langle \xi_m, \xi_1 \rangle \right| \Bigg|_{M = N^2 + 2N}$$

$\left\{ \begin{array}{l} |\cdot| \text{ is done} \\ \text{componentwise} \\ (\infty\text{-norm?}) \end{array} \right.$

$$\leq \frac{1}{N} \sum_{m=2}^{N^2+2N} \left| \xi_m \, \Omega_m \left[1 + \sqrt{2(N+1)} \right] \right|$$

$$\leq \frac{|\Omega_{max}|}{N} \cdot \left(1 + \sqrt{2(N+1)} \right) \cdot \sum_{m=2}^{N^2+N} (1)$$

$$\approx \frac{|\Omega_{max}|}{\cancel{N}} \cdot \sqrt{2N} \cdot \cancel{N}(N+1)$$

Case 1: Say $\Omega_i = O(1) \quad \forall\, i$

$\Omega_i \neq \Omega_j \quad \forall\, i \neq j$

$\Omega_i < 0 \quad \forall\, i$

$$\Omega_1 - \frac{\sum_{m=1}^{M} \Omega_m}{N} \approx \frac{M\, O(1)}{N} - O(1)$$

$$= \frac{(N^{\cancel{2}} + 2\cancel{N})\, O(1)}{\cancel{N}} - O(1)$$

$$= (N+2)\, O(1) - O(1)$$