Exercise 1. Give a formal and precise definition for the maximization version of an approximation-ratio preserving polynomial reduction.

Exercise 2 (All pairs shortest path)**.** Suppose we have a graph with weighted edges given by its adjacency matrix. For example

$$M = \begin{pmatrix} 0 & 1 & 4 & \infty \\ 1 & 0 & 2 & \infty \\ 4 & 2 & 0 & 1 \\ \infty & \infty & 1 & 0 \end{pmatrix}$$

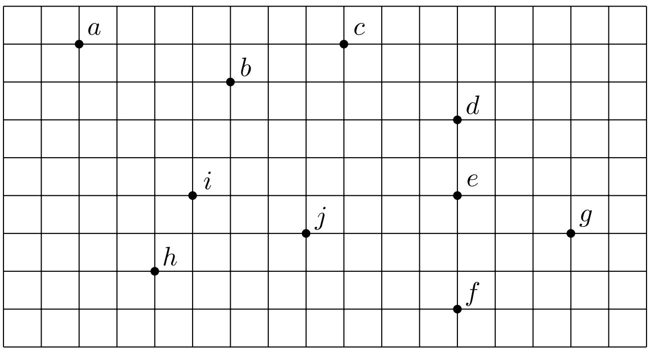
We want to compute the shortest path between all pairs, or equivalently the adjacency matrix of the metric closure of the graph.

- A first algorithm is to compute powers of the matrix M where the usual addition is replaced by a min and the usual multiplication is replaced by a sum.
  - What does the matrix M^i represent in this setting ?
  - What power do we need to take to be sure to have computed the all pairs shortest path ? What is the complexity of this algorithm ?
  - Apply the algorithm on the matrix given at the beginning.
- A faster approach, known as Floyd-Warshall algorithm, can be implemented by this code on a $n \times n$ matrix:

```
for k=1..n do
    for i=1..n do
        for j=1..n do
            M[i][j] = min( M[i][j], M[i][k] + M[k][j] )
```

  Give its complexity and explain why we obtain the same result at the end. Apply it to the matrix given at the beginning.

Exercise 3. Consider this set of points in the plane where the distance between points is given by the usual Euclidean distance:



1. Apply the factor-2 approximation algorithm to compute a Steiner tree that passes through the vertices $\{a, d, h, i, g\}$.
2. Apply the TSP factor-2 approximation algorithm seen in class. Start the Euler tour from a and turn clockwise.
3. Apply the TSP factor-3/2 approximation algorithm seen in class. We will start the Euler tour from a and turn clockwise.

Exercise 4. Show that $|\{(x_1, \ldots, x_k) \in \mathbf{N}_0^k \mid \sum x_i \leq m\}| = \binom{m+k}{m}$.