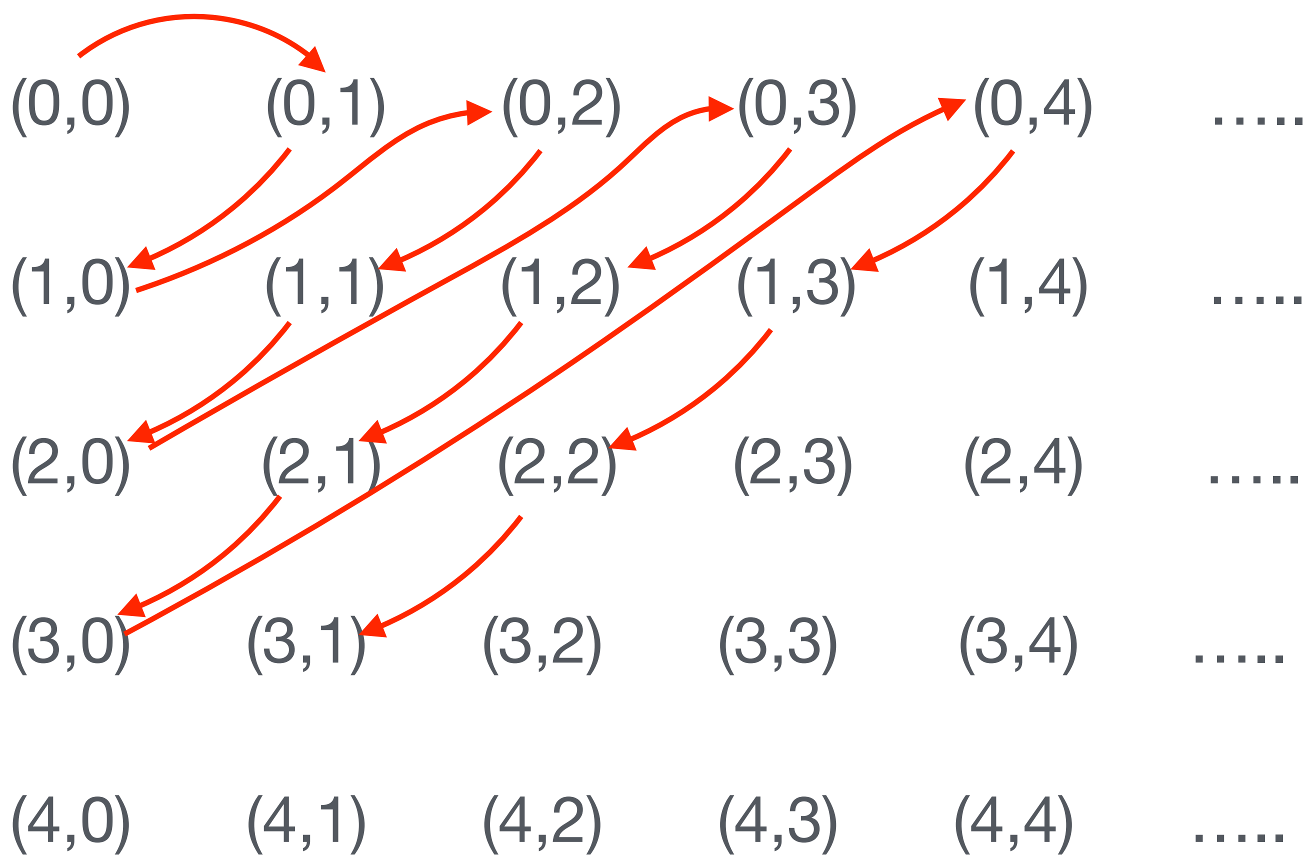
Countability of *N* x *N*

- In the class, we looked at the following way to count the elements of *N* x *N*



$(0,0) \longmapsto 0$	0+0
$(0,1) \longmapsto 1$	1+0
$(1,0) \longmapsto 2$	1+1
$(0,2) \longmapsto 3$	(1+2)+0
$(1,1) \longmapsto 4$	(1+2)+1
$(2,0) \longmapsto 5$	(1+2)+2
$(0,3) \longmapsto 6$	(1+2+3)+0
etc.	

$$(a,b) \longmapsto (1+2+3+\ldots+(a+b)) + a =: f(a+b)+a$$

What we Need to Show

$\mathbf{N} \times \mathbf{N} \longrightarrow \mathbf{N}$, $(a,b) \longmapsto f(a+b)+a$, is injective!

- Suppose that it is not injective.
- Then $f(a+b)+a = f(x+y)+x$ for two pairs $(a,b)\neq(x,y)$
- Assume first that $(a+b)\neq(x+y)$.
  - If $(a+b) < (x+y)$
    - Then $f(a+b)+a = (1+2+\ldots+(a+b))+a < (1+2+\ldots+(a+b))+a+b+1 = f(a+b+1) \leq f(x+y)+x$, which is a contradiction to $f(a+b)+a = f(x+y)+x$.
  - Similarly, $(a+b) > (x+y)$ leads to a contradiction.
- Therefore, $(a+b) = (x+y)$.
  - $f(a+b)+a = f(x+y)+x$ then implies $a=x$, and since $(a+b) = (x+y)$, also $b=y$, so $(a,b)=(x,y)$, a contradiction.
- So, the mapping is injective, and we are done!