Exercise 1. Consider the linear program

$$\begin{aligned} \text{maximize} \quad & x_1 - 2x_3 \\ \text{subject to} \quad & x_1 - x_2 \leq 1 \\ & 2x_2 - x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Prove that for this program the solution $(x_1, x_2, x_3) = (1.5, 0.5, 0)$ is optimal.

Exercise 2. Consider the following "minimax" problem:

$$\begin{aligned} \text{minimize} \quad & \max_{i \in I} x_i \\ \text{subject to} \quad & \mathcal{A}x = b, \\ & x \geq 0, \end{aligned}$$

where x is an n-dimensional column vector with real entries and I is a fixed subset of indices in $\{1, \ldots, n\}$.

1. Express this problem as a linear program.
2. Let P be a set of n points in $\mathcal{R}^2$. We want to find a line that minimizes the maximum vertical distance to the points in P (the vertical distance between a line $y = ax + b$ and a point (x_0, y_0) is defined as $|ax_0 + b - y_0|$). Give a linear program for this problem.

Exercise 3. Suppose that, given a directed graph G, two distinct vertices s and t, and positive weights on the edges, we want to find the *shortest path* from s to t. Formulate this problem as a linear program and obtain its dual.