2-Antenna Diagonal Space-Time Codes and Continued Fractions

$$\frac{u}{L} = \cfrac{1}{q_1 + \cfrac{1}{q_2 + \cfrac{1}{q_3 + \cfrac{1}{\cdots + \cfrac{1}{q_{t-1} + 1}}}}}$$

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Outline

In this talk we will show that the best diversity distance for 2-antenna diagonal space-time codes is obtained if the number of signals is a Fibonacci number.

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

Mobile Communication Unknown Channel

Signal set: A finite set $\mathcal{S} = \{S_1, S_2, \ldots, S_L\}$ of diagonal unitary $M \times M$-matrices (M = number of transmit antennas).

Use differential encoding for transmission.

Pairwise probability of error is smaller the larger the diversity distance

$$\zeta(\mathcal{S}) := \min_{S,R \in \mathcal{S}} |\det(S - R)|^{1/M}$$

is. Want to maximize this distance.

2-Antenna Codes

$$\mathcal{S}(u, L) = \left\{ \begin{pmatrix} \eta^k & 0 \\ 0 & \eta^{ku} \end{pmatrix} \,\middle|\, 0 \leq k < L \right\}, \qquad \eta = \mathrm{e}^{2\pi i/L}.$$

Diversity distance is

$$\zeta_{u,L} = \min_{1 \leq k < L} \frac{1}{2} |1 - \eta^k||1 - \eta^{ku}| = 4 \left| \sin\left(\frac{k\pi}{L}\right) \sin\left(\frac{ku\pi}{L}\right) \right|.$$

Define $\zeta_L := \max_{1 \leq u < L} \zeta_{u,L}$.

For given L determine u such that $\zeta_{u,L} = \zeta_L$.

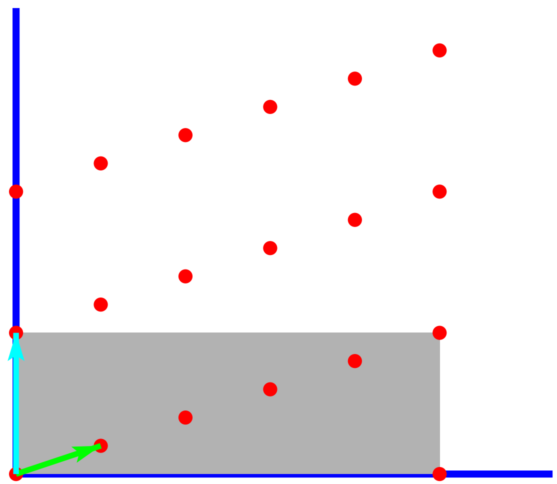
Conjecture

$\zeta_{F_n} = \zeta_{F_n, F_{n-2}}$, where F_n is the nth Fibonacci number.

Lattices

$\Lambda(u, L) := \mathbb{Z}(0, L) + \mathbb{Z}(1, u)$.

We prove that for fixed L, $\zeta_{u,L}$ is proportional to $\rho(u, L)$, the area of smallest rectangle inside $\Lambda(u, L)$. (Uses ideas from Clarkson et al.)



Continued Fractions

$$\frac{u}{L} = \cfrac{1}{q_1 + \cfrac{1}{q_2 + \cfrac{1}{q_3 + \cfrac{1}{\cdots + \cfrac{1}{q_{t-1} + 1}}}}}$$

The q_i are called partial quotients.

Gives rise to the sequence of convergents

$$\frac{P_1}{Q_1}, \quad \frac{P_2}{Q_2}, \quad \ldots, \quad \frac{P_t}{Q_t} = \frac{u}{L}.$$

Convergents and the Smallest Area Rectangle

$$\rho(u,L) = \min_{\ell} LQ_\ell^2 \left| \frac{u}{L} - \frac{P_\ell}{Q_\ell} \right|.$$

The worse u/L can be approximated by its convergents, the larger is $\rho(u,L)$.

The smaller the partial quotients of u/L are, the worse u/L can be approximated by its convergents.

Fibonacci Numbers

The partial quotients of F_{n-2}/F_n consist entirely of 1's.

$$\max_{1 \le u < F_n} \rho(u, F_n) = \rho(F_{n-2}, F_n).$$

Unfortunately, this does not necessarily mean that $\zeta_{F_n} = \zeta_{F_{n-2},F_n}$, but we conjecture that this is the case.

Further Remarks

- For general L, it is best to choose a u such that u/L has smallest sequence of partial quotients (in L_1-norm).

- No polynomial time algorithm known to compute u from L.

- This would minimize $\rho(u, L)$, but not necessarily $\zeta_{u,L}$ (counterexamples exist).

- Our techniques could be used to design good codes from reducible representations of the Quaternion groups.