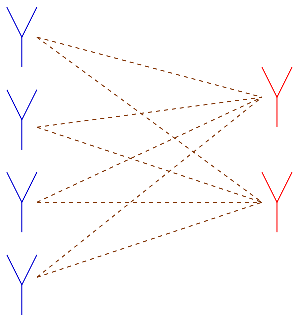
Computing the Performance of Group Space Time Codes From their Character Table



Amin Shokrollahi

Outline

Want to compute upper bounds on the pairwise probability of error unitary differential space-time codes that form a finite group under matrix multiplication.

Will do so by using the character table of the group.

Transmission: Rayleigh Flat Fading

M transmit antennas, N receiving antennas, coherence interval T.

$$\begin{pmatrix} s_{T,1} & s_{T-1,1} & \cdots & s_{1,1} \\ s_{T,2} & s_{T-1,2} & \cdots & s_{1,2} \\ \vdots & \vdots & \ddots & \vdots \\ s_{T,M} & s_{T-1,M} & \cdots & s_{1,M} \end{pmatrix} =: S.$$

Received signal:

$$X := \underbrace{\sqrt{\rho}}_{\text{SNR}} \cdot \underbrace{S}_{\text{Signal}} \cdot \underbrace{H}_{\text{Fading}} + \underbrace{W}_{\text{Noise}},$$

where H is $T \times N$ and W is $M \times N$ and entries are independent $CN(0,1)$ random variables.

Decoding: Compute S from X.

Codebook Modulation

$\mathcal{S} = \{S_1, S_2, \ldots, S_L\}$.

String $(e_0, \ldots, e_{\ell-1})$ corresponds to

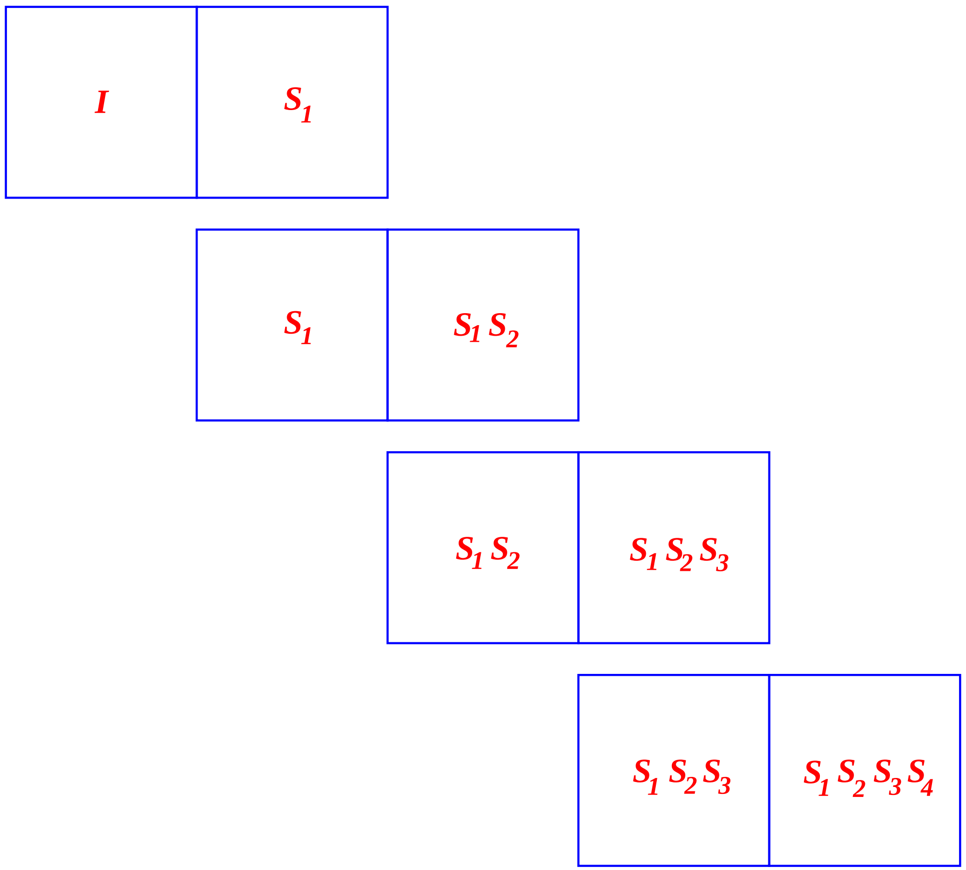
$$S_i, \qquad i = e_0 + 2e_1 + \cdots + 2^{\ell-1} e_{\ell-1}.$$

Simple encoding possible if presentation of $\mathcal{S}$ is appropriate.

Mobile Communication: H is unknown

Use differential encoding.

$T = 2M$.



Differential Encoding

Codebook consists of L unitary $M \times M$-matrices $\{S_1, S_2, \ldots, S_L\}$ and is called a unitary space-time code.

Signals transmitted:

$$S_{i_1}, S_{i_1}S_{i_2}, S_{i_1}S_{i_2}S_{i_3}, \ldots.$$

H is eliminated:

$$X = HS + W, \qquad Y = HSR + W, \qquad XS + \tilde{W} = Y.$$

Unknown Channel: Decoding and Probability of Error

Maximum likelihood decoding: given $X, Y \in \mathbb{C}^{N\times M}$, find $S \in \mathcal{S}$ that minimizes

$$||XS - Y||$$

for some matrix norm $||\cdot||$.

Probability $P(S,R)$ of mistaking S for R

$$P(S,R) \leq \frac{1}{2}\prod_{m=1}^{M}\left[1 + \frac{\rho^2}{4(1+2\rho)}\sigma_m^2(S-R)\right]^{-N},$$

where ρ is the SNR.

Description in terms of Characteristic Polynomial

$$P(S, R) \leq \frac{1}{2}\left(\frac{8}{\rho}\right)^{N \deg \tilde{T}(x)} \cdot \tilde{T}(1)^{-2N},$$

- $T(x)$ is the characteristic polynomial of SR^*,
- $T(x) = \tilde{T}(x)(x-1)^k$,
- $\tilde{T}(1) \neq 0$.

Group Case

- If signals form a finite group under matrix multiplication, then SR^* belongs to the set.

- Pairwise probability of error depends only on conjugacy class of SR^*.

- The values of $P(S, R)$ depend on the conjugacy classes of the group.

Is there an efficient way to compute these values using as little data as possible about the underlying group? Yes!

Character Table

The character table of a group tabulates for any irreducible representation D of the group and for any conjugacy class C of the group the sum of the eigenvalues of $D(C)$.

Example: The symmetric group S_3

	C_1	C_2	C_3
id	1	1	1
sgn	1	-1	1
χ	2	0	-1

Power Maps and Newton Relations

- Also need the power map function: Which conjugacy class is the mth power of a given class?

- Using this and the character table, we know the values of sums of mth powers of the eigenvalues of SR^*

- Can use Newton relations to compute the characteristic polynomial.

- Very efficient!

The Groups $SL(2, p)$

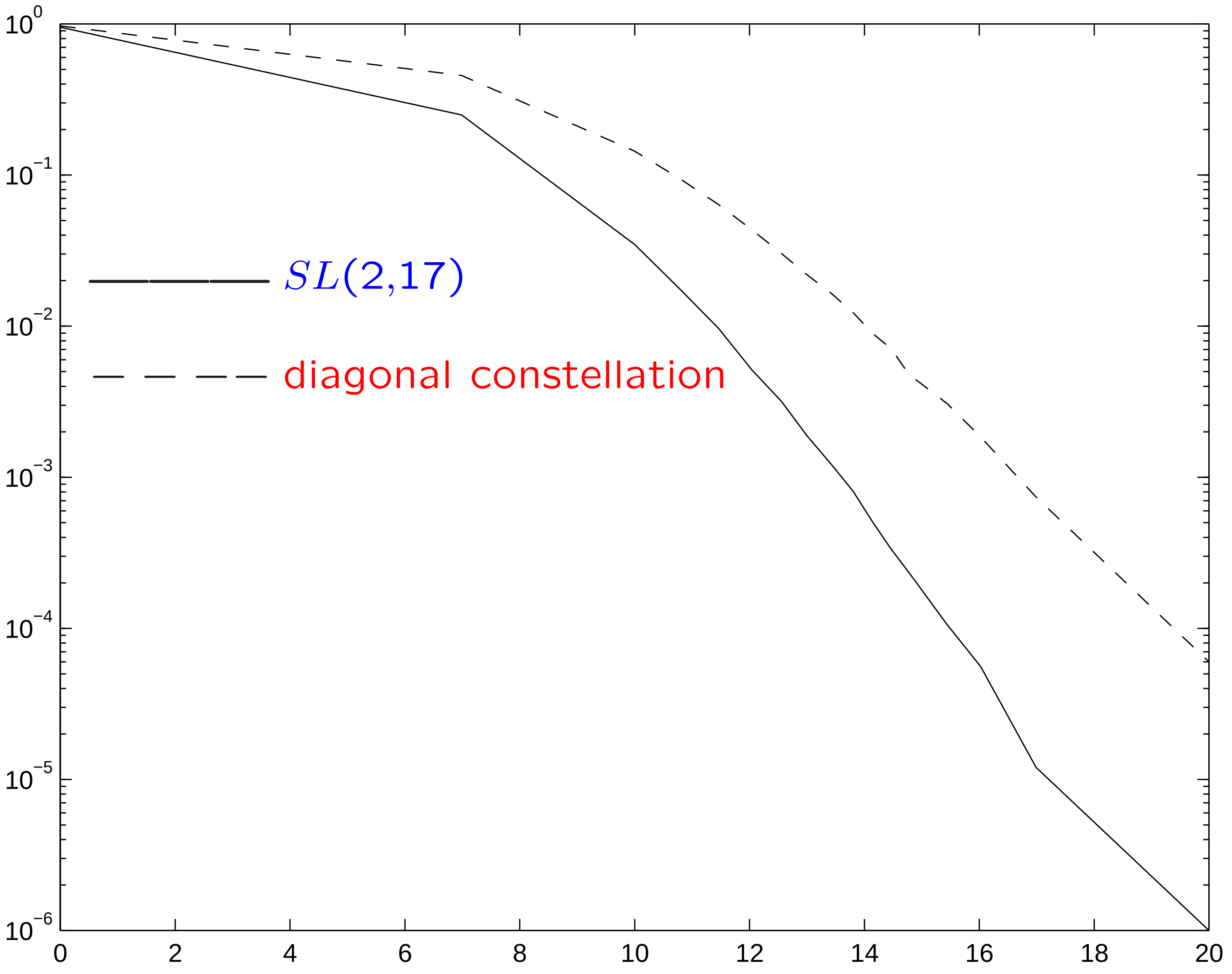
- Full paper computes characteristic polynomials for all irreducible representations of all the groups $SL(2, p)$ for odd prime p.

- $SL(2, 3)$ and $SL(2, 5)$ are fixed-point-free (already classified).

- 8dimensional irreducible representation of $SL(2, 17)$ is not fixed-point-free but has very good behavior with respect to pairwise error probability.

- Gives constellation with 8 transmit antennas and rate 1.53.

$SL(2,17)$



10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
0
2
4
6
8
10
12
14
16
18
20
SL(2,17)
diagonal constellation

Conclusions

- To look for good space-time group codes, we need to compute representations of groups (difficult).

- Using the character table we can efficiently discard bad groups and concentrate on good ones.

- Once good groups are found, we can compute their corresponding representations to obtain the desired space-time codes.